Exam Seat No:_______________ Enrollment No:__________________

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code : 5SC02MTC1 **Summer Examination-2014** **Date: 9/06/2014**

Subject Name Differential Geometry

Branch/Semester:- M.Sc(Mathematics) /II **Time:02:00 To 5:00**

Examination: Regular

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary

(2) Use of Programmable calculator & any other electronic instrument is prohibited.

(3) Instructions written on main answer Book are strictly to be obeyed.

(4) Draw neat diagrams & figures (If necessary) at right places

(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1 a) Prove that $\bar{t}' \cdot \bar{b}' = -k\tau$. (02)

b) Find tangent plane for the surface $xyz = a^3$ where a is constant. (02)

c) Define spherical indicatrix of tangent. (02)

d) If $\bar{r}$ is a plane curve with no where vanishing curvature then what is the value of τ. (01)

Q-2 a) Find curvature and torsion of the curve $\bar{r} = (u - \sin u, 1 - \cos u, 3)$. (05)

b) Let $\bar{r}$ be an arc length parameterized curve. Find tangent, principal normal, binormal, curvature and torsion of the locus of centre of the locus of centre of spherical curvature of $\bar{r}$. (05)

c) Prove that a curve is helix if ratio of curvature and torsion is constant. (04)

OR

Q-2 a) Let $\bar{r}(t)$ be a curve and $\dot{r}(t) \neq 0$ for all t then prove that curvature $k = \dfrac{|\dot{r} \times \ddot{r}|}{|\dot{r}|^3}$ (05)

b) State and prove fundamental theorem of curve theory. (05)

c) Define involutes. Find tangent, principal normal and curvature of an Involutes of a given curve. (04)

Q-3 a) Prove that the envelope of the family of paraboloids $x^2 + y^2 = 4a(z - a)$ is $x^2 + y^2 = z^2$. (05)

b) Prove that the surface $xy = (z - c)^2$ is a developable. (05)

c) Find first and second fundamental magnitudes of surface $\bar{r} = (u\cos\phi, u\sin\phi, f(\phi))$. (04)

OR

Q-3 a) Derive necessary condition for surface $z = f(x, y)$ representing a developable surface. (05)

b) Find edge of regression of the envelop of the family of planes $x\sin\theta - y\cos\theta + z\theta = a\theta$ where θ is parameter. (05)

c) Show that sum of square of intercepts on the co – ordinate axis made by the tangent plane to the surface $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$. (04)

SECTION-II

Q-4 a) Write equation for principal direction. (02)

b) Define flat point. (02)

c) Define lines of curvature. (02)

d) What is the value of k_n for asymptotic line? (01)

Q-5 a) If k_a, k_b be the principal curvature and k_n denote the normal curvature at a point on the surface and Ψ be the angle between the direction (du, dv) and the principal direction $v =$ constant (05)

$k_n = k_a \cos^2\Psi + k_b \sin^2\Psi$.

b) If k is a curvature of plane section of a surface $S: \bar{r} = \bar{r}(u, v)$ and k_n is a curvature of a normal section at the same point. Let θ be an angle between principal normal to the curve obtain by plane section at a point P and $\bar{n}$ then $k_n = k\cos\theta$. (05)

c) Check whether the parametric curves for surface $\bar{r} = (u\cos\emptyset,\ u\ \sin\emptyset,\ \cos^2 u)$ are conjugate or not. (04)

OR

Q-5 a) Prove that equation for principal curvature is $H^2k^2 - (EN - 2FM + GL)k + T^2 = 0$. Also write formula for Gaussian curvature. (05)

b) The necessary and sufficient condition for lines of curvature to be parametric curve is $F = 0 = M$. (05)

c) Find asymptotic lines and its torsion for the surface generated by tangents of a twisted curve. (04)

Q-6 a) Find Christoffel symbols of the first kind for the surface $\bar{r} = (u\cos\phi,\ u\ \sin\phi,\ f(u))$. (05)

b) Prove that all points on the surface $\vec{r} = (a\cos\theta\cos\varphi, a\cos\theta\sin\varphi, a\sin\theta)$ are elliptic points. (05)

c) The geodesic torsion τ_g at a point P on a curve C is $\frac{1}{2}(k_a - k_b)\sin 2\varphi$, where φ is the angle between geodesic tangent and $v =$ constant. (04)

OR

Q-6 a) Prove that Christoffel symbol of first kind $\Gamma_{ijk} = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial u^i} + \frac{\partial g_{ki}}{\partial u^j} - \frac{\partial g_{ij}}{\partial u^k}\right)$. (05)

b) If k_g denote the geodesic curvature, prove that $k_g = [\bar{N}\bar{r}'\bar{r}'']$ (05)

c) Let τ and τ_g denote the usual torsion and geodesic torsion at a point P on a curve C on the surface and ω denote the positive angle between $\bar{n}$ and surface normal $\bar{N}$, prove that $\tau_g = \tau + \frac{d\omega}{ds}$. (04)

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